Abstract

In this brief position paper I introduce a generalisation of the classical labour theory of value that establishes a lawful relation between the prices of reproducible commodities and the labour time required to produce them.

1. The problems of the classical theory

The problems of the classical labour theory of value manifest in multi-sector input-output models with “inequality of the proportions in which labour and means of production are employed in the various industries” (Sraffa, 1960, Ch. 3).

Consider a steady-state economy:

Definition 1. A steady-state economy produces (i) quantities, $q = qA^T + w + c$, where $A$ is a productive input-output matrix, and $w = [w_i]$ and $c = [c_i]$ are worker and capitalist consumption bundles, at (ii) prices, $p = (pA + lw)(1 + \pi)$, where $\pi$ is the profit rate, $l$ are direct labour coefficients and $w$ is the wage rate, where (iii) workers and capitalists spend what they earn, $pw^T = lq^Tw$ and $pc^T = (pA + lw)q^T\pi$.

In the steady state, workers produce a net product, $n = w + c$, which is distributed and consumed within the period of production. Over multiple periods the economy self-replaces with a constant composition and scale.

Marx defined the value of a commodity as the socially necessary labour time embodied in means of production plus the new labour added by the worker (Marx, [1867] 1954). The modifier “socially necessary” controls for variable labour productivity within sectors. Since this does not apply here we discuss it no further.

Definition 2. Classical labour values, $v = vA + l$, are the sum of the value of means of production, $vA$, and new labour added, $l$.

Marx’s theory of value proposes that natural prices, $p$, are reducible to some function of labour values, $v$ (Marx, [1894] 1971, pgs. 157–8). In special circumstances, such as zero profit, prices are proportional to labour values. But in general, labour values are a function of the techniques of production alone (i.e., $A$ and $l$) whereas prices are also a function of the distribution of income (i.e., $\pi$ and $w$). Prices have an additional degree-of-freedom unrelated to labour values and therefore may vary independently of them. In consequence labour values cannot fully explain the structure of natural prices, even in the simple case of a steady-state economy (Wright, 2014a).

This explanatory gap creates various problems for the classical labour theory of value, most notably Ricardo’s problem of an invariable measure of value and Karl Marx’s transformation problem (e.g., see Wright (2014a); Seton (1957); Desai (1988); Hunt & Glick (1990)).

Many authors therefore conclude that the labour theory of value is, at best, incomplete, or worse, logically incoherent, e.g. Samuelson (1971); Lippi (1979); Steedman (1981).

But this conclusion is premature, as we now demonstrate.

2. Classical subsystems

Sraffa (1960, p. 89) proposed to decompose an economy into “as many parts as there are commodities in its net product, in such a way that each part forms a smaller self-replacing system the net product of which consists of only one kind of commodity. These parts we shall call ‘subsystems’.” A subsystem is a vertically-integrated ‘slice’ of the economy that produces one unit of a single commodity as net output and replaces the used-up means of production.

Define the ‘subsystem matrix’, $(I - A)^{-1}$. The matrix inverse operation is equivalent to vertical integration. The $i$th column of the subsystem matrix therefore represents the activity levels of the Sraffian subsystem that produces commodity $i$.

Classical labour values, in terms of subsystems, are $v = l(I - A)^{-1}$ (a consequence of definition 2). A commodity’s labour value is therefore the total coexisting labour

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(Marx, 2000, Ch. 21) supplied to its vertically integrated subsystem. A labour value sums all the labour, supplied in parallel to all the different sectors of the economy, that cooperate to produce the commodity. Interpreting labour values as the coexisting labour supplied to subsystems is clarifying. For example, following Pasinetti (1980, pg. 21), we can partition the length of the working day in two ways: as (i) the sum of direct labour supplied to each sector of production, \( \sum l_i q_i = l q^T \), or (ii) the sum of the coexisting labour supplied to each Sraffian subsystem, \( \sum v_i n_i = v n^T \). The partitions are quantitatively equal, that is \( l q^T = v n^T \), because the Sraffian subsystems collectively produce the net product as final output and exhaust the total supplied labour:

**Proposition 1.** The total working day equals the classical labour value of the net product, \( l q^T = v n^T \).

**Proof.** \( q = q A^T + n = n (I - A^T)^{-1} \). Hence \( l q^T = (I - A)^{-1} n^T = v n^T \). \( \square \)

The working day is composed of the direct labour supplied to each sector that produces the total product. But it is also composed of the coexisting labour supplied to each subsystem that produces the net product.

### 3. Generalised subsystems

Pasinetti (1988) develops more general multi-sector input-output models that feature net investment in means of production to expand the economy at non-uniform growth rates. In this context, Pasinetti defines hyper-subsystems that produce 1 unit of a commodity as final output, replaces the means of production and replaces the net investment goods.

Pasinetti also defines a generalisation of classical labour values – which he calls the “vertically hyper-integrated labour coefficients” – that equal the total coexisting labour supplied to each hyper-subsystem.

The point is this: different vertical integration policies (e.g., whether to include investment goods) define different kinds of subsystem (e.g., Sraffian or hyper subsystems) and different kinds of labour values (e.g., classical labour values and hyper-integrated labour coefficients). The different kinds of subsystems are alternative, but complementary, methods of partitioning a multi-sector economy (for more details see Wright (2017)).

In consequence, a commodity has a classical labour value and a hyper-integrated coefficient. Which to use? That depends on the question posed. For example, a classical labour value will undercount the coexisting labour necessary to produce a commodity while simultaneously growing the scale of its production.

### 4. Super-integrated subsystems

We now introduce a new kind of subsystem. Define a vertically super-integrated subsystem as a Sraffian subsystem augmented by the production that replaces capitalist consumption goods.

To construct the \( i \)th commodity’s super-integrated subsystem we need to know what capitalists consume during its production.

The quantity of commodity \( i \) consumed by capitalist households per unit of profit income is \( c_i / (p A + lw) q^T \pi \) where \( c_i \) is the total consumption of commodity \( i \) and \( (p A + lw) q^T \pi \) is total profit income. The profit income received by capitalist households, per unit output in sector \( j \), is \( (p A^{(j)} + l_j w) \pi \), where \( A^{(j)} \) is the \( j \)th column of matrix \( A \). Hence, consumption coefficient \( \alpha_{i,j} = c_i (p A^{(j)} + l_j w) / ((p A + lw) q^T) \) denotes the quantity of commodity \( i \) distributed to capitalist households per unit output of \( j \), where \( (p A^{(j)} + l_j w) / ((p A + lw) q^T) \) is a dimensionless ratio that denotes the proportion of profit generated in sector \( j \).

Define \( C = [\alpha_{i,j}] \) as the capitalist consumption matrix. \( C \) compactly describes the flow rate of consumption goods to capitalist households per unit output of each commodity.

We deduced matrix \( C \) via the price system. Nonetheless, \( C \) is a ‘physical’ consumption matrix, independent of the price system, and solely determined by real properties of the economy, as we now demonstrate.

First, note that the profit rate is determined by the technique and the real distribution of income (i.e., the ‘break down of the net product’ \( n \) into worker, \( w \), and capitalist consumption, \( c \)):

**Lemma 1.** In a steady-state economy the rate of profit, \( \pi \), is the dominant eigenvalue of matrix \( (I - A - W)(A + W)^{-1} \), where \( W = (1/l q^T) w^T l \) is the worker consumption matrix.

**Proof.** Let \( X = (I - A - W)(A + W)^{-1} \). Then \( p X = \lambda p \) is an eigenvalue equation. Solve the characteristic equation, \( \det(X - \lambda I) = 0 \), to obtain \( \lambda = \lambda^* \), where \( \lambda^* \) is the Perron-Frobenius root (dominant eigenvalue). Multiply the eigenvalue equation by \( (A + W) \) to obtain \( \lambda p (A + W) = p - p A - p W \), which implies \( p = p (A + W) (1 + \lambda^*) \).

Since \( W = (1/l q^T) w^T l \), then \( p = (p A + lw) (1 + \lambda^*) \), which are the prices of the steady-state economy. Hence, \( \pi = \lambda^* \). \( \square \)

In consequence:

**Proposition 2.** The capitalist consumption matrix, \( C \), is a function of the technique, \( A \) and \( I \), and the real distribution of income, \( w \) and \( c \).
The general theory of labour value

Proof. Define the capitalist consumption matrix,

\[ C^* = \frac{c^T(I - A(1 + \pi))^{-1}(A + W)}{l(A(1 + \pi))^{-1}(A + W)(w + c)^T(I - A)^{-1}}. \]

The profit-rate, \( \pi \), by lemma 1, is a function of \( A \) and worker consumption matrix \( W \). Matrix \( W \), by definition, is a function of \( I \), \( w \) and quantities, \( q \). And \( q \), by definition 1, is a function of \( A \), \( w \) and \( c \). Hence, \( C^* \) is a function of the real properties of the economy only.

We now show that \( C^* \) is equivalent to \( C \). Note that \( p/(w(1 + \pi)) = l(I - A(1 + \pi))^{-1} \) and \( q^T = (w + c)^T(I - A)^{-1} \) (from definition 1). Substitute into equation (1) to yield \( C = c^T p(A + W)/p(A + W)q^T = c^T(pA + lw)/(pA + lw)q^T = [a_{i,j}]. \)

The total output of the \( i \)-th super-integrated subsystem is

\[ q_i = u_i + \hat{q}_i A^T + \hat{q}_i C^T, \]

where \( u_i \) is a zero vector except for the \( i \)-th component that equals 1. The \( i \)-th vertically super-integrated subsystem consists of the direct \( (u_i) \), indirect \( (\hat{q}_i A^T) \) and super-indirect \( (\hat{q}_i C^T) \) production that produces 1 unit of \( i \) as final output, where “super-indirect” refers to the production of capitalist consumption goods.

Sraffian subsystems are defined by purely technological conditions alone. In contrast, super-integrated subsystems capture the institutional fact that production, in a capitalist system, materially reproduces the real income of a capitalist class.

5. Super-integrated labour coefficients

Every kind of subsystem implicitly defines a kind of labour value.

Definition 3. The vertically super-integrated labour coefficients, \( \tilde{v} \), for a steady-state economy, are

\[ \tilde{v} = 1 + \tilde{v} A + \tilde{v} C, \]

which is the sum of direct, indirect and super-indirect labour costs.

The coefficients, \( \tilde{v} \), represent the total coexisting labour supplied to each super-integrated subsystem; or, in Marx’s terminology, they count the labour time embodied in means of production and capitalist consumption goods plus the new labour added.

The definition of the super-integrated coefficients does not provide or rely upon any theory of income distribution or profit. However, in order to calculate prices, the distribution of nominal income must be given datum. Conjectural variation in equilibrium of either the real or nominal distribution of income then affects both the super-integrated coefficients and prices.

The super-integrated labour coefficients directly relate, in a straightforward manner, to the total working day.

Proposition 3. The total working day equals the super-integrated labour value of the real wage, \( \tilde{v} w^T = l q^T \).

Proof. \( C q^T = (1/(pA + lw)q^T)c^T ((pA + lw)q^T) = c^T. \) Substitute for \( q^T \) into the quantity equation to yield, \( q = q A^T + q C^T + w = w(l - A^T - C^{-1}w). \) Hence \( l q^T = l(I - A - C)^{-1}w = \tilde{v} w^T. \)

In consequence, we can partition the working day into the sum of direct, indirect and super-indirect labour supplied to each super-integrated sector, \( \sum q_i w_i = \tilde{v} w^T. \) Again, this partition is quantitatively equal to the total labour supplied, that is \( l q^T = \tilde{v} w^T \), because the super-integrated subsystems collectively produce the real wage as final output and exhaust the total supplied labour.

We now have two measures of labour value: classical and super-integrated. Classical values are a special case of super-integrated values in circumstances of zero profit (since \( \tilde{v} = \tilde{v}(A + C) + 1 \) reduces to \( \tilde{v} = \tilde{v} A + 1 = v \) in circumstances of zero capitalist consumption, \( C = 0 \)).

The classical definition ignores, whereas the super-integrated definition includes, the labour cost of reproducing the income of a capitalist class.\(^1\) In this sense, classical values are counterfactual whereas super-integrated values are actual measures of the labour supplied to produce commodities in the institutional conditions of a capitalist economy.

Both measures are needed to address the full range of issues raised by a labour theory of value.

6. Prices as labour values

We now state the main result of this paper.

Theorem 1. The prices of a steady-state economy are proportional to the super-integrated labour coefficients, \( p = \tilde{v} w. \)

Proof. By definition 1 capitalists spend what they earn, \( (pA + lw)q^T \pi = pc^T \). Substitute for \( \pi \) into the price equation: \( p = pA + (pA + lw) \pi + lw = pA + (pA + lw)(pc^T/(pA + lw)q^T) + lw = pA + (1/(pA + lw)q^T)c^T(pA + lw) + lw = p(A + C) + lw = l(I - A - C)^{-1}w = \tilde{v} w. \)

\(^1\)Note that any definition of labour value is necessarily independent of the real wage; see Wright (2015, pg. 50–52).
The price of a commodity is the wage bill of the total coexisting labour required to reproduce it. Commodities that cost more labour time to produce sell at proportionally higher prices in equilibrium. The more general definition of labour value replicates the result, established by Adam Smith for an “early and rude state” of society, that ‘labour embodied’ equals ‘labour commanded’.

The classical authors believed that competitive prices diverged from labour values due to “profits on stock”. This premise has been universally accepted, both by supporters and critics of the labour theory of value. However, the divergence arises because classical labour values undercut the coexisting labour supplied to produce commodities in the institutional conditions of a capitalist economy. Once we count the actual coexisting labour supplied, that is the super-integrated labour values, there is no divergence. The essential duality between the nominal and real cost structures of an economy is then revealed.

Theorem 1, it should be emphasised, removes the logical basis for rejecting the labour theory of value based on a supposed ineradicable mismatch between prices and labour values (e.g., (Samuelson, 1971; Lippi, 1979; Steedman, 1981)).

7. A general theory

The more general theory, sketched here, admits both technical and social measures of labour cost and applies them in the appropriate context.

For example, classical labour values apply to distribution-independent questions about an economy, such as measuring the technical productivity of labour (e.g., Flaschel (2010, part 1)) or the surplus labour supplied gratis by workers to the capitalist class (e.g., Marx ((1867) 1954)); whereas the super-integrated labour coefficients apply to distribution-dependent questions, such as the relationship between prices and the actual labour time supplied to produce commodities; i.e., issues in the theory of value.

The general theory dissolves the classical contradictions, such as Ricardo’s problem of an invariable measure of value and Marx’s transformation problem (Wright, 2014a), identifies a deeper structural contradiction in Marx’s theory of capitalism (Wright, 2015, Ch. 3), provides a new and clarifying interpretation of Sraffa’s standard commodity in terms of super-integrated labour values (Wright, 2015, Ch. 4) and dissolves Pasinetti’s “general transformation problem” that manifests in his non-uniform growth model (Wright, 2017).

8. A dynamic, multi-sector input-output model

The classical authors, such as Smith and Marx, explained economic coordination as the unintended consequence of the self-interested decisions of economic actors engaged in competition (e.g., Smith’s “invisible hand” or Marx’s “law of value”). Capitalists, who seek the best returns on their investments, withdraw capital from unprofitable sectors and reallocate it to profitable sectors. The scramble for profit eliminates arbitrage opportunities until a general or uniform profit-rate prevails across the whole economy, at which point capitalists lack any incentive to reallocate their capital. Capitalist competition, according to the classical authors, is a mechanism that causes market prices of reproducible commodities to gravitate toward or around their competitive, or natural, prices (e.g., Smith ((1776) 1994), Book 1, Chapter VII).

Theorem 1 implicitly assumes gravitation has operated to completion. We now drop this assumption and construct a nonlinear, multi-sector input-output model of classical gravitation that reconstructs Marx’s “law of value” (Wright, 2015, Ch. 7), which establishes a lawful relation between market prices and labour values.

8.1. Worker households

Assume constant returns to scale. The composition of the real wage, \( \mathbf{w} \), is constant but may vary in scale, such that

\[
w(t) = \frac{\alpha \mathbf{m}_w(t)}{\mathbf{p}(t) \mathbf{w}^\top} \mathbf{w},
\]

where \( \alpha \in (0, 1] \) is workers’ propensity to consume and \( \mathbf{m}_w(t) \) is their money stock. The fraction \( \alpha \mathbf{m}_w(t)/\mathbf{p}(t) \mathbf{w}^\top \) is the quantity of real wage bundles of composition \( \mathbf{w} \) purchased.

The change in workers’ money stocks is the difference between income and expenditure:

\[
\frac{\mathrm{d}m_w(t)}{\mathrm{d}t} = \mathbf{l}(t)^\top \mathbf{w}(t) - \alpha \mathbf{m}_w(t).
\]

The change in the wage rate depends on both the level and rate of change of employment:

\[
\frac{\mathrm{d}w(t)}{\mathrm{d}t} = \eta \mathbf{l}(t)^\top \frac{1}{\mathbf{L} - \mathbf{l}(t)^\top} \mathbf{w}(t),
\]

where \( \eta > 0 \) is an elasticity coefficient and \( \mathbf{L} \) is the size of the available work force (this is a Philips-like labour market (1958)).
8.2. Capitalist households

Capitalist consumption is
\[ c(t) = \frac{\alpha_c m_c(t)}{p(t)c^*_t}, \]
where \( c \) is its composition, \( \alpha_c \) is capitalists’ propensity to consume and \( m_c(t) \) is their money stock.

The change in capitalists’ money stocks is the difference between income and expenditure. The capitalist sector has two sources of income: interest from loans that finance production and industrial profit (or loss) from the ownership of firms.

Assume that firms in sector \( i \) finance their costs of production – by borrowing money-capital from finance capitalists. Finance capitalists receive interest payments on the money-capital currently ‘tied-up’ in production on a continuous (‘daily’) basis, at a varying, instantaneous interest rate \( r(t) \). The interest income from sector \( i \) is \( \kappa_i(t)r(t) \) and therefore the aggregate interest income is
\[ \sum_{i=1}^{n} \kappa_i(t)r(t) = (p(t)A + lw(t))q(t)^T r(t). \]

Industrial capitalists, as owners of firms, are subject to profit and loss, which is the difference between firms’ costs and revenue.

Firms in sector \( i \) have aggregate costs, \( \kappa_i(t) \), plus interest payments to service the debt to money-capitalists, \( \kappa_i(t)r(t) \). Total production costs are therefore \( \kappa_i(t)(1 + r(t)) \).

The demand for commodity \( i \), \( d_i(t) \), consists of demand from other sectors and households, \( d_i(t) = A_{ij}q_j(t)^T + w_i(t) + c_i(t) \). The total revenue, generated by firms in sector \( i \), is the price of the sold output, \( p_i(t)d_i(t) \).

We can now construct a profit function. The current profit (or loss) in sector \( i \) is the difference between total revenue and total cost; that is
\[ \pi_i(t) = p_i(t)d_i(t) - \kappa_i(t)(1 + r(t)). \]  
(4)

Capitalists households include both finance and industrial capitalists. The aggregate money stock, \( m_c(t) \), is augmented by an inflow of profit – consisting of total interest income and total industrial profits – and reduced by an outflow of consumption spending and total industrial losses. The change in money stock is therefore
\[ \frac{dm_c(t)}{dt} = \sum_{i=1}^{n} \kappa_i(t)r(t) + \sum_{i=1}^{n} \pi_i(t) - \alpha_c m_c(t). \]  
(5)

8.3. The interest rate

Marx, in Volume 3 of Capital, adopts a loanable funds theory of the interest rate. For simplicity assume the stock of loanable funds is the total money stock held by workers and capitalists.

**Proposition 4.** The total money stock in the economy is constant, \( m_w(t) + m_c(t) = m_w(0) + m_c(0) = M \).

**Proof.** Note that \( \sum_{i=1}^{n} \pi_i(t) = \alpha_w m_w + \alpha_c m_c - (pA + lw)q^T r \) and \( \frac{dm_w}{dt} + \frac{dm_c}{dt} = -\alpha_w m_w - \alpha_c m_c + lq^T w + (pA + lw)q^T r + \sum_{i=1}^{n} \pi_i(t) \). Combine to get \( \frac{dm_w}{dt} + \frac{dm_c}{dt} = 0 \). Hence \( m_w(t) + m_c(t) = k \), where \( k \) is a constant of integration. At \( t = 0 \) we have \( k = m_w(0) + m_c(0) \). \( \square \)

In this example the available loanable funds are a fixed constant which implies a fixed interest rate, \( r(t) = r(0) \). Note, however, that \( M \) may turn over multiple times to support very different quantities of outstanding debt.

8.4. Firms

Firms buy inputs and hire in labour to produce output that is sold in the market. They strategically adjust the prices they charge and the quantities they produce in response to market conditions.

8.4.1. Inventories

Supply in general does not equal demand. In consequence, each sector stores a stock of unsold inventory, denoted \( s_i(t) \). The change of inventories equals excess supply; that is,
\[ \frac{ds_i(t)}{dt} = q_i(t) - d_i(t). \]  
(6)

8.4.2. Price adjustment

Firms raise prices when inventories shrink since buyers outbid each other to obtain the scarce product, whereas firms lower prices when inventories grow since firms underbid each other to sell to scarce buyers. The sector as a whole adjusts relative price in proportion to excess demand, that is \( \frac{dp_i}{p_i} \propto -\frac{ds_i}{dt} \). This has a cross-dual form: a quantity imbalance, represented by the change in inventory size, translates into a price adjustment.

Assume the change in price approaches positive \( \infty \) as inventory approaches zero and the commodity is completely scarce, that is \( \frac{dp_i}{p_i} \propto \frac{1}{s_i} \). Combining these two factors we get the price adjustment equation
\[ \frac{dp_i(t)}{dt} = -\eta_i \frac{ds_i(t)}{dt} \frac{p_i(t)}{s_i(t)}. \]  
(7)

where \( \eta_i \) is an elasticity coefficient.
8.4.3. Output adjustment

Industrial capitalists adjust their production plans based on profit and loss. A firm that returns a profit (resp. loss) borrows more (resp. less) money in the market for loanable funds in order to increase (resp. decrease) supply with the expectation of earning greater profit (resp. reducing losses).

Industrial capitalists, as a whole, own a portfolio of firms grouped into sectors that, at any time, make different profits or losses. The profit rate in sector $i$, $\pi_i/(\kappa_i(1 + r))$ is the ratio of profit to costs.

Capitalists aim to maximise their profit by differentially injecting or withdrawing capital based on profit rate signals. The relative change in the scale of production is therefore proportional to the profit rate, that is $1/\eta_{i+n} \propto \pi_i/(\kappa_i(1 + r))$. This has a cross-dual form: a price imbalance, represented by the profit rate, translates into a quantity adjustment. Define the quantity adjustment equation

$$1/\eta_{i+n} \frac{dq_i(t)}{dt} = \pi_i(t)/(\kappa_i(t)(1 + r(t))), \quad (8)$$

where $\eta_{i+n} > 0$ is an elasticity constant. Sectors with a high (resp. low) profit rate increase (resp. reduce) their borrowing in order to increase (resp. decrease) the supply of goods to the market.

8.5. A system of nonlinear, ordinary differential equations

The eqs. (2), (3) and (5) to (8) combine and simplify to a $(2n + 1)$-dimensional “classical macrodynamic” system of nonlinear, ordinary differential equations in prices, $p(t)$, quantities, $q(t)$, and workers’ savings, $m_w(t)$ with $3n + 3$ initial conditions $(p(0), q(0), s(0), m_w(0), w(0), r(0))$, $2n + 2$ elasticity parameters $(\eta = [\eta_1], \eta_m, \eta_w)$, a given stock of money $M$ and available labour force $L$ (Wright, 2015, Ch. 7).

Systems of nonlinear differential equations yield closed form solutions only in special cases. So we use numerical methods to solve them. To explore this model you can download an interactive simulator (see Wright (2014b)).

9. Classical macrodynamics

The classical macrodynamics are rich and varied. To give some idea we’ll briefly examine a small, 3-sector economy, and focus on the trajectories of prices, quantities and the wage rate (and ignore other issues, such as employment and the distribution of income).

Consider an economy that produces corn, iron and sugar, with parameters

$$A = \begin{bmatrix} 0.02 & 0 & 0.01 \\ 0.2 & 0.5 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$1 = [0.7, 0.6, 0.3], \quad w = [0.6, 0.2]$$

(workers consume corn and sugar but not iron), $c = [0.2, 0.4]$ (capitalists proportionally consume more sugar than corn compared to workers), $p(0) = [1, 0.5], \quad q(0) = [0.1, 0.1, 0.1], \quad s(0) = [0.1, 0.1, 0.1], \quad w(0) = 0.5, \quad r(0) = 0.03, \quad m_w(0) = m_c(0) = 0.5$ (worker and capitalist savings are initially equal and the total money stock in the economy is $M = 1$), $\alpha_w = 0.8$ and $\alpha_c = 0.7$ (workers have a higher propensity to consume), $L = 1$, the price elasticities are $\eta_1 = \eta_2 = \eta_3 = 2$, the quantity elasticities are $\eta_4 = \eta_5 = \eta_6 = 1$, the wage is relatively inelastic, $\eta_w = 0.25$.

These parameters instantiate an economy that follows a growth trajectory until it reaches a steady-state equilibrium where prices, quantities and the distribution of income are constant over time. See Figure 1 for a description of the trajectories.

Numerical simulations demonstrate that the macrodynamic system is locally asymptotically stable. I have proved stability in a special case (Wright, 2011) but the problem remains open for the general case.

The macrodynamic model demonstrates that the classical theory of gravitation is a successful and logically coherent explanation of the homoeostatic kernel of capitalist competition. The model supports Garegnani’s (1990) and Serrano’s (2011) view that classical competition through capital mobility may ensure gravitation under quite general conditions (Bellino & Serrano, 2011).

10. Competitive equilibrium

The macrodynamic system converges to the steady state economy of definition 1. So the dynamic nonlinear model embeds linear production theory at its equilibrium point as a special case. We briefly indicate this result by deriving the equilibrium price and quantity solutions.

First, note that, in equilibrium, the industrial profit rate is uniformly zero (see Figure 1(c)).

Lemma 2. Profit-of-enterprise is uniformly zero in equilibrium, $\pi_i = 0$ for all $i$.

Proof. Substitute $\frac{dq}{dt} = 0$ into quantity adjustment equation (8) to get $\pi_i = 0$ for all $i$. \qed

Industrial profit, in a fully competitive system, is a disequilibrium phenomenon. Profit attracts (and repels) capital investment. But the scramble for profit has the unintended
The general theory of labour value

(a) **Prices.** The price of corn and sugar initially rises (due to relative scarcity) then falls (as output adjusts). The price of iron mainly falls (since it’s over produced during gravitation). Prices stabilise at $t \approx 10$.

(b) **Quantities.** The scale of production increases in all sectors (since all sectors are initially profit making), with some overshoot in the sugar sector. Quantities stabilise at $t \approx 10$.

(c) **Profits.** All sectors are initially but differentially profitable. Capitalists increase the scale of production. Profits fall as supply starts to meet demand. At $t \approx 5$ the iron and sugar sectors are loss making. So capital withdraws and their production decreases. At $t \approx 10$ industrial profits are uniformly zero across all sectors.

(d) **Wage rate.** The economy grows and employs more of the available labour force. The labour market tightens and wages rise. At $t \approx 10$ the economy reaches a steady state equilibrium and the wage stabilises at $w \approx 0.85$.

Figure 1. Classical gravitation in a 3-sector economy. Sectors: corn (black), iron (dashed), sugar (dots).

Consequence of reducing imbalances between supply and demand, which ultimately eliminates arbitrage opportunities and causes profit to fall.

**Proposition 5.** Equilibrium prices in terms of the equilibrium wage, $w^*$, and interest rate, $r(0)$, are

$$p^* = (p^*A + l w^*) (1 + r(0)), \quad (9)$$

where

$$w^* = k_w \frac{1}{(L - l q^*)^\eta w}.$$

**Proof.** Apply the zero profit condition of Lemma 2 to profit function 4 to obtain $p_i^* d_i^* = \kappa_i (1 + r(0))$ for all $i$. Inventory adjustment equation (6), with $\frac{d q_i^*}{d t} = 0$, implies $q_i = d_i$. Hence, $p_i^* q_i^* = \kappa_i (1 + r^*)$. Expand, simplify, and write in vector form. Solve wage adjustment equation (3) to obtain the wage rate as a function of the level of employment, where $k_w = w(0)(L - l q(0))^\eta w$ is a positive constant.

**Proposition 6.** Equilibrium quantities in terms of the equilibrium net product, $n^*$, are

$$q^* = n^*(I - A^T)^{-1} \quad (10)$$

where

$$w^* = \frac{\alpha w^m}{p^* w^T w}$$

is the equilibrium real wage and

$$c^* = \frac{\alpha c (M - m^*_w)}{p^* c^T c} \quad (11)$$

is the equilibrium real consumption of capitalists.
The general theory of labour value

(a) The classical law of value. Prices divided by classical labour values get close to, but do not converge to, the equilibrium wage rate (cf. Ricardo’s 93% labour theory of value).

(b) The general law of value. Prices divided by super-integrated labour values approach and equal the equilibrium wage rate.

Figure 2. The law of value in a 3-sector economy. Sectors: corn (black), iron (dashed), sugar (dots).

Proof. Set \( \frac{ds_i}{dt} = 0 \) in equation (6) to get \( q_i = d_i \) for all \( i \). Expand, simplify and write in vector form to yield the conclusion.

Equations (9) and (10) are structurally equivalent to the price and quantity equations of definition 1, except profit is composed entirely of interest income. The macrodynamic system is a monetary model that determines absolute prices; in contrast, linear production models are undetermined up to a choice of an arbitrary numéraire and therefore only determine relative prices.

The macrodynamic equilibrium is entirely independent of initial prices, \( p(0) \), initial inventories, \( s(0) \), the initial distribution of money wealth, \( m_w(0) \) and \( m_c(0) \), and price and quantity adjustment elasticities, \( \eta_i \) for \( i \in [1, 2n] \) (for a full specification, see Wright (2015, pgs. 179–181)). As might be expected, out-of-equilibrium scarcity prices of reproducible commodities turn out to be irrelevant to the equilibrium state. Scarcity prices are transient phenomena that quickly dissipate as production is reorganised to meet final demand – they affect the trajectory toward equilibrium but not the equilibrium itself.

This model, although classical in inspiration, shares many features with Post Keynesian economic analysis (Rogers, 1989, Ch. 7). One example: there is a “limit to the profitable expansion of output” (Chick, 1983, p. 71) before full employment is reached.

11. The general law of value

The “law of value” is a theory of the coordination of social labour time via out-of-equilibrium mismatches between the labour-embodied in, and labour-commanded by, commodities (see especially Rubin (1973) and Pilling (1986)).

Smith, Ricardo and Marx all restrict the applicability of the law of value in various ways. Smith ([1776] 1994, Ch. 6) restricts the law to pre-civilised times. Ricardo notes that income distribution is a “less powerful cause” Ricardo (2005, p. 404) of competitive prices and proposed what has become known as the “93% labour theory of value” (Stigler, 1958). Marx ([1894] 1971, p. 158) claims that the law applies but empirically manifests in a distorted form due to capitalist property relations. The more general theory, adopted here, offers a new perspective.

Marx’s version of the law of value seeks to establish two claims: first, the causal claim that, in appropriate conditions, prices are determined, and therefore explained, by real costs of production measured in labour time; and, second, the semantic claim that “labour is the substance, and the immanent measure of value” (Marx, [1867] 1954, p. 503) in the sense that monetary phenomena refer to, express, or measure labour time in virtue of a lawful, one-to-one relation that obtains between them.

The causal claim – that prices are ultimately determined by classical labour values – fails because prices, in general, are also determined by class conflict over the distribution of income.

The semantic claim – that classical gravitation establishes a lawful relationship between prices and labour costs – holds because gravitation is simultaneously a process by which the nominal and real cost structures of the economy grope toward a state of mutual consistency.

Figure 2 illustrates this process. It plots the trajectory of market prices divided by their labour values (both classical and super-integrated) during convergence to equilibrium (for the same 3-sector economy). Prices deflated by their classical labour values approach but do not equal the
wage rate (consistent with Ricardo’s 93% theory and the observed empirical correlation between input-output prices and classical labour values; see for example Shaikh & Tonak (1994); Cockshott et al. (1995); Zachariah (2006); Fröhlich (2013)). But prices do not bear a lawful, one-to-one relation to classical labour values because varying the interest rate, \(r(0)\), alters the correlation.

In contrast, prices deflated by their super-integrated labour values approach and eventually equal the wage rate in equilibrium, at which point Theorem 1 applies (see Wright (2015, sec. 7.3.2)). Prices bear a lawful, one-to-one relation to super-integrated labour values. In consequence, the dynamics of capitalist competition instantiate a causal regularity, or law, that causes the ‘labour embodied’ in commodities to equal the ‘labour commanded’ by them, at which point the allocation of labour to the different sectors of production perfectly matches the final demand for commodities.

The “general law of value” affects the trajectory of market prices at all times. But the equilibrium state could only fully manifest under conditions of fast gravitation compared to slow changes in propensities to consume, elasticities, and composition of demand, plus a constant technique that is relatively invariant to changes in scale.

12. Conclusion

The immediate experience of the personal benefit derived from market exchange motivates subjective theories of value. However, market exchange has causal consequences beyond the immediate moment that derive from its embodiment within a system of general commodity production.

The modern separation of the classical surplus approach from its labour theory of value does not constitute a sophisticated rejection of naive “substance” theories of value but indicates a failure to resolve the classical contradictions. Post-Sraffian classical economics therefore dispenses with an essential aim of a theory of economic value, which is to explain what the unit of account may represent. Theorem 1, which demonstrates that equilibrium prices are proportional to physical quantities of labour, starts to put the pieces back together again.

Marx proposed to formulate the economic laws that explain why monetary magnitudes represent labour time (just as the law of thermal expansion explains why the height of a mercury column represents temperature). The macrodynamic system, which constitutes a minimal formalisation of the dynamics of capitalist competition, supports Marx’s proposition that “labour is the substance, and the immanent measure of value” (Marx, [1867] 1954, p. 503) by establishing that the prices of reproducible commodities refer to labour time in virtue of a general law of value that binds them.

Appendix

Numerical example of Theorem 1

The technique and the distribution of real income determines the super-integrated labour values and competitive prices, which are dual to each other.

For example, consider a steady-state, 2-sector economy with technique

\[
A = \begin{bmatrix} 0 & 0.5 \\ 0.25 & 0 \end{bmatrix},
\]

direct labour coefficients, \(1 = [1, 2]\), and worker and capitalist consumption bundles, \(w = [1, 0.05]\) and \(c = [0.5, 0.1]\).

First, we calculate the super-integrated labour values. Quantities \(q = (w + c)(I - A^T)^{-1} = [1.8, 0.6]\). Worker consumption matrix

\[
W = \frac{1}{lq} w^T l = \begin{bmatrix} 0.33 & 0.66 \\ 0.017 & 0.033 \end{bmatrix}.
\]

The profit rate, by Lemma 1, is the dominant eigenvalue of \(\det((I - A - W)(A + W)^{-1} - \pi I) = 0\), i.e.,

\[
\begin{vmatrix} -(1.11 + \pi) & 3.89 \\ 0.89 & -(2.11 + \pi) \end{vmatrix} = 0,
\]

which yields \(\pi = 0.31\). The capitalist consumption matrix, by equation (1), is

\[
C = \begin{bmatrix} 0.18 & 0.29 \\ 0.036 & 0.058 \end{bmatrix}.
\]

The super-integrated labour values are then \(\bar{v} = l(I - A - C)^{-1} = [2.77, 4.45]\).

Second, we calculate competitive prices. \(p = l(I - A(1 + \pi))^{-1}(1 + \pi)w = [2.77, 4.45]w\).

Hence, prices are proportional to super-integrated labour values, \(p = \bar{v}w\), as per Theorem 1.

References


